[2nd Kyu]   Section 2: Application Test

1 (Selective)

A sector has a perimeter 20 cm and its central angle is less than $360°$. Find the range of the radius of the sector such that the range of the area, A, is given by

$16 \text{ cm}^2 \leq A \leq 24 \text{ cm}^2$.

2 (Selective)

You play a game using a die numbered 1 to 6. You roll the die three times in a row. When the same number is on the top face three times, you get 3 points. When the same number is on the top face exactly two times in a row, that is, on the 1st and 2nd rolls or 2nd and 3rd rolls, you get 2 points. For the other cases, you get 1 point.

(1) Find the probability that you get 2 points in this game.

(2) Find the expected value of your points when you play the game.

3 (Selective)

Let P and Q be the points of intersection between the circle $x^2 + y^2 = 5$ and the straight line $x + y = 1$, where the value of the x-coordinate of point P is less than that of point Q.

(1) Find the coordinates of points P and Q.

(2) Let R be the point of intersection between the tangent line to the circle at P and the tangent line to the circle at Q. Find the coordinates of point R.

4 (Selective)

Consider a sequence $\{a_n\}$ whose sum of the first number to the n th number is expressed as $S_n = n(n+1)(n+2)$.

(1) Find the n th term, a_n, of the sequence $\{a_n\}$.

(2) Find the following sum.

$$\sum_{n=1}^{99} \frac{1}{a_n}$$

5 (Selective)

Find all sets of integers, (x, y, z), satisfying the following equality. Write only your answer. (*Organizing skill*)

$$x^4 + y^4 + z^4 = 3xyz$$

6 (Required)

Consider the following proposition for integer n. *(Proof skill)*

"If $n^2 + 2n$ is an odd number, then n is an odd number."

(1) Give the contrapositive of the proposition. Write only your answer.

(2) Prove that the proposition is true using your answer in (1).

7 (Required)

Let p be the parabola $y = -x^2 + 3x$ and let ℓ be the straight line $y = 2x$.

(1) Find the coordinates of the points of intersection between the parabola p and the straight line ℓ.

(2) For the area bounded by the parabola p and the x-axis, find the area of the figure below the straight line ℓ.