1

Let x cm be the radius of the sector. The length of the arc can be expressed as $(20-2x)$ cm.

Since the central angle is less than $360°$, the length of the arc is less than the circumference of the circle, so we have

$$0<20-2x<2\pi x$$

Hence $\frac{10}{\pi+1}<x<10$ …①

The area of the sector is

$$\frac{1}{2}\times x\times(20-2x)=x(10-x)$$

$$=-x^2+10x(\mathrm{cm}^2)$$

So

$$-x^2+10x\geq 16 \quad \cdots②$$

$$-x^2+10x\leq 24 \quad \cdots③$$

From ②

$$x^2-10x+16\leq 0$$

$$(x-2)(x-8)\leq 0 \quad \therefore \quad 2\leq x\leq 8 \quad \cdots②'$$

From ③

$$x^2-10x+24\geq 0$$

$$(x-6)(x-4)\geq 0 \quad \therefore \quad x\leq 4,\ 6\leq x \quad \cdots③'$$

The required range is the intersection of ①, ②' and ③'. So we have

$$\frac{10}{\pi+1}<x\leq 4,\quad 6\leq x\leq 8$$

(Note that $2=\frac{10}{5}<\frac{10}{\pi+1}$)

(Answer) $\frac{10}{\pi+1}$ cm $<x\leq 4$ cm or

(Answer) 6 cm $\leq x\leq 8$ cm

2

(1) The total number of ways when rolling a die 3 times is $6^3=216$ (ways)

The number of ways you get 2 points is

- the same number is on the top face on the 1st and 2nd rolls or the 2nd and 3rd rolls, i.e. 2 ways.
- the number of ways of the same number is 6 ways.
- the number of ways of the other number is 5 ways.

Hence

$$2\times 6\times 5=60 \text{ (ways)}$$

Therefore, the probability is

$$\frac{60}{216}=\frac{5}{18}$$

(Answer) $\frac{5}{18}$

(2) The probability that you get 3 points is

$$\frac{6}{216}=\frac{1}{36}$$

The probability that you get 1 point is

$$1-\frac{60}{216}-\frac{6}{216}=\frac{150}{216}=\frac{25}{36}$$

Hence the expected value of your point is

$$1\times\frac{25}{36}+2\times\frac{5}{18}+3\times\frac{1}{36}$$

$$=\frac{25+20+3}{36}=\frac{48}{36}=\frac{4}{3}$$

(Answer) $\frac{4}{3}$

3

(1) The coordinates of the points of intersection are the solution of the following system of equations：

$$\begin{cases} x^2+y^2=5 & \cdots① \\ x+y=1 & \cdots② \end{cases}$$

From ②, we have $y=1-x$, substitute it into ①,

$$x^2+(1-x)^2=5$$

$$x^2-x-2=0$$

$$(x-2)(x+1)=0 \quad \text{So} \quad x=-1,\ 2$$

When $x=-1$, $y=2$

When $x=2$, $y=-1$

Hence $\mathrm{P}(-1,\ 2)$ and $\mathrm{Q}(2,\ -1)$.

(Answer) $\mathrm{P}(-1,\ 2)$, $\mathrm{Q}(2,\ -1)$

(2) The equation of the tangent line at point $\mathrm{P}(-1,\ 2)$ is

$$-x+2y=5 \quad \cdots③$$

The equation of the tangent line at point $\mathrm{Q}(2,\ -1)$ is

$$2x-y=5 \quad \cdots④$$

From ③＋④×2,

$$3x=15 \quad \text{So} \quad x=5$$

Substitute it into ③, we get $y=5$

Therefore $\mathrm{R}(5,\ 5)$.

(Answer) $\mathrm{R}(5,\ 5)$

4

(1) When $n=1$, $a_1=S_1=1\cdot 2\cdot 3=6$ …①

When $n\geq 2$

$$a_n=S_n-S_{n-1}$$
$$=n(n+1)(n+2)-(n-1)n(n+1)$$
$$=n(n+1)\{(n+2)-(n-1)\}$$
$$=3n(n+1)\ \cdots②$$

If $n=1$ in ②, the result is equal to ①.

② is satisfied when $n=1$.

Therefore the nth term is $a_n=3n(n+1)$.

(Answer) $a_n=3n(n+1)$

(2) From (1), we have

$$\frac{1}{a_n}=\frac{1}{3n(n+1)}=\frac{1}{3}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

So

$$3\times\sum_{n=1}^{99}\frac{1}{a_n}$$
$$=\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots$$
$$+\left(\frac{1}{99}-\frac{1}{100}\right)$$
$$=1-\frac{1}{100}=\frac{99}{100}$$

Hence $\sum_{n=1}^{99}\frac{1}{a_n}=\frac{33}{100}$

(Answer) $\frac{33}{100}$

5

(Answer) $(0,\ 0,\ 0)$, $(1,\ 1,\ 1)$, $(1,\ -1,\ -1)$, $(-1,\ 1,\ -1)$, $(-1,\ -1,\ 1)$

6

(1) (Answer) If n is an even number, then n^2+2n is an even number.

(2) If n is even, it can be expressed as $n=2k$, where k is an integer. Then we have

$$n^2+2n=4k^2+4k=2(2k^2+2k)$$

Since $2k^2+2k$ is an integer, n^2+2n is an even number. Therefore, since the contrapositive is true, the proposition is also true.

7

(1) The x-coordinates of the points of intersection of the parabola $p: y=-x^2+3x$ and the straight line $\ell: y=2x$ are found by

$$-x^2+3x=2x$$
$$x^2-x=0$$
$$x(x-1)=0$$

Hence $x=0$ or 1. Substitute them into the equation ℓ and the coordinates the intersections are

$(0,\ 0)$ and $(1,\ 2)$

(Answer) $(0,\ 0)$, $(1,\ 2)$

(2) The x-coordinates of the parabola p and the x-axis is found by

$$-x^2+3x=x(3-x)=0$$

Hence $x=0$ or 3.

When $0\leq x\leq 1$, $0\leq 2x\leq -x^2+3x$.

When $1\leq x\leq 3$, $0\leq -x^2+3x\leq 2x$.

The required area is

$$\int_0^1(2x)dx+\int_1^3(-x^2+3x)dx$$
$$=\Big[x^2\Big]_0^1+\left[-\frac{x^3}{3}+\frac{3}{2}x^2\right]_1^3$$
$$=1-0+\left(-9+\frac{27}{2}\right)-\left(-\frac{1}{3}+\frac{3}{2}\right)$$
$$=\frac{13}{3}$$

(Answer) $\frac{13}{3}$