1

(1) (Answer) OP$=5$ cm, PH$=\sqrt{21}$ cm

(2) Using the Pythagorean theorem for $\triangle$PAH,

$$PA^2=PH^2+AH^2$$
$$=21+9=30$$

Since PA>0, PA$=\sqrt{30}$ (cm)

Using the Pythagorean theorem for $\triangle$PBH, we have

$$PB^2=PH^2+BH^2$$
$$=21+49=70$$

Since PB>0, PB$=\sqrt{70}$ (cm).

Hence PA : PB$=\sqrt{30}:\sqrt{70}$

$$=\sqrt{3}:\sqrt{7}$$

(Answer) $\sqrt{3}:\sqrt{7}$

2

(3) Two consecutive odd numbers can be expressed as

$2n-1$, $2n+1$. (n is an integer)

Their difference is

$$(2n+1)^2-(2n-1)^2$$
$$=4n^2+4n+1-(4n^2-4n+1)$$
$$=8n=4\times 2n$$

Here, $2n$ is the even number between the two odd numbers.

Therefore, for two consecutive odd numbers, the difference between the square of them equals 4 times of the even number between them.

3

(4) (Answer) $a=-30$, the other root: $x=-6$

4

(5) (Answer) $(20-2x)$ cm

(6) Since the length of the arc is positive and less than the circumference of the circle,

$$0<20-2x<2\pi x.$$

Hence, $\frac{10}{1+\pi}<x<10$ …①

Let θ° be the central angle of the sector,

$$\frac{\theta}{360}=\frac{20-2x}{2\pi x}=\frac{10-x}{\pi x}$$

Let S (cm^2) be the area of the sector,

$$S=\pi x^2\times\frac{10-x}{\pi x}$$
$$=x(10-x)=-x^2+10x$$
$$=-(x-5)^2+25$$

So, S takes the maximum value 25 at $x=5$ in the interval ①.

(Answer) The maximum area is 25 cm^2 when the radius is 5 cm.

5

(7) The total number of ways possible outcomes is

$$6\times6\times6=216 \text{ (ways)}$$

There are 6 ways for the 1st and 2nd rolls. For each of the 6 ways, there are 5 ways for the 3rd roll.

So, the probability is

$$\frac{6\times5}{216}=\frac{30}{216}=\frac{5}{36}$$

(Answer) $\frac{5}{36}$

6

(8) (Answer) $\frac{3\sqrt{15}}{4}$

(9) Since $\sin A=\frac{\sqrt{15}}{4}$,

$$\cos^2 A=1-\sin^2 A$$
$$=1-\frac{15}{16}=\frac{1}{16}$$

Since $90°<A<180°$,

$$\cos A=-\frac{1}{4}$$

Using the law of cosines for $\triangle$ABC,

$$BC^2$$
$$=AB^2+AC^2-2\,AB\cdot AC\cdot\cos A$$
$$=2^2+3^2-2\cdot2\cdot3\cdot\left(-\frac{1}{4}\right)$$
$$=4+9+3$$
$$=16$$

Since BC>0

BC$=4$

(Answer) BC$=4$

7

(10) (Answer) $(x, y, z)=(0, 0, 0)$, $(1, 1, 1)$, $(1, -1, -1)$, $(-1, 1, -1)$, $(-1, -1, 1)$