Pre 1st Kyu

Section 2 : Application Test

数学検定

PROFICIENCY TEST
IN
PRACTICAL MATHEMATICS

Test Time : 120 minutes

Test Instructions

1 . Make sure that you have the correct level (Kyu) test.
2 . Do not open the booklet until you are told to do so.
3 . Write your examinee number and name on this page.
4 . Write your name, examinee number and other necessary information on the answer sheets.
5 . Write your answers on the answer sheets (they are numbered 1 through 4). Write the steps leading to your answer. However if there are specific instructions for a problem, follow the instructions.
6 . Problems 1 to 5 are selective problems. Choose two problems from the selective problems and fill in ○ to indicate which problems you chose. Then write your answers. Note that all of your answers will not be marked if you answered more than two problems from the selective problems. Problems 6 and 7 are required problems.
7 . You may use a calculator.
8 . Turn off your cell phone and do not use it during the test.
9 . Ask an examination supervisor if your problem sheets have inconsistent page numbering or missing pages.
10. It is prohibited to disclose the problems to the general public, such as on the Internet, without permission.

Examinee Number	–	Name	

※Your personal information will be handled appropriately according to the "Handling of Personal Information" agreement that was approved at the time of registration.

公益財団法人
日本数学検定協会
The Mathematics Certification Institute of Japan

[Pre-1st Kyu] Section 2: Application Test

1 (Selective)

Let m be a constant. Determine the value of m such that the following equation represents two lines in the xy-plane.

$$x^2 + 2xy - y^2 + 2mx + (6-m)y + 4 = 0$$

2 (Selective)

A sequence $\{a_n\}$ of positive numbers satisfies

$$a_1 = 1, \quad a_2 = 1, \quad \frac{a_n a_{n+1} - 2a_n a_{n+2} + a_{n+1} a_{n+2}}{a_n a_{n+1} a_{n+2}} = n \quad \text{for } n = 1, 2, 3, \ldots.$$

(1) Letting $b_n = \dfrac{1}{a_{n+1}} - \dfrac{1}{a_n}$, express $b_{n+1} - b_n$ in terms of n. (*Expression skill*)

(2) Find the nth term, denoted by a_n, of the sequence $\{a_n\}$.

3 (Selective)

Let a, b, c and d be real constants with $c^2+d^2 \neq 0$. The functions $f(x)$ and $g(x)$ are defined by

$$f(x)=\frac{x+1}{2x+3} \text{ and } g(x)=\frac{ax+b}{cx+d}.$$

(1) Find $g(x)$ if $f(g(x))=x+1$ is an identity in x.

(2) Find $g(x)$ if $g(f(x))=x+1$ is an identity in x.

4 (Selective)

Point P(x, y) is represented by the following pair of parametric equations with parameter t, $t>0$.

$$x=3+\frac{4t}{t^2+1} \text{ and } y=\frac{1-t^2}{t^2+1}$$

(1) Find the range of values of x. Write only your answer.

(2) Find the locus of point P.

5 (Selective)

$f(n)$ is defined by the following for a positive integer n, where k is an integer.

(i)	If n is even that is represented by $2k$, $f(n)=3k$.
(ii)	If n is odd that is represented by $n=4k+1$, $f(n)=3k+1$.
(iii)	If n is odd that is represented by $n=4k+3$, $f(n)=3k+2$.

Given a positive integer n, we obtain a positive integer N by $N=f(n)$. We call this procedure "operation C." We replace N with n and perform operation C repeatedly. For example, starting with $n=17$, the number 12 is obtained by performing operation C six times as follows:

$$17 \xrightarrow{\text{(ii)}} 13 \xrightarrow{\text{(ii)}} 10 \xrightarrow{\text{(i)}} 15 \xrightarrow{\text{(iii)}} 11 \xrightarrow{\text{(iii)}} 8 \xrightarrow{\text{(i)}} 12$$

Answer the following. (*Organizing skill*)

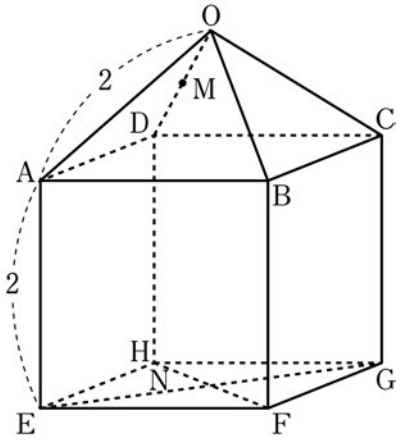
(1) Confirm that there is only one positive integer n such that $N=f(n)$ for an arbitrary positive integer N.

(2) For two distinct positive integers a and b, if a leads to b by performing operation C several times or if b leads to a by performing operation C several times, we say "a and b are on the same orbit."
Among the one-digit positive integers excluding 8, how many numbers are on the same orbit as 8? Give your reason.

(3) A positive integer m leads to 8 after performing operation C ten times. Find the value of m. Write only your answer.

6 (Required)

The figure shows a solid composed of right square pyramid OABCD with edges of length 2 and cube ABCD-EFGH with edges of length 2, sharing face ABCD. Let M be the midpoint of line segment OD and let N be the point of intersection of diagonals of square EFGH.



Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = \vec{c}$.

(*Expression skill*)

(1) Express $\overrightarrow{OD}$ and $\overrightarrow{ON}$ in terms of $\vec{a}$, $\vec{b}$ and $\vec{c}$.

(2) Let P be the point of intersection of line MN and plane ABC. Express $\overrightarrow{OP}$ in terms of $\vec{a}$, $\vec{b}$ and $\vec{c}$.

7 (Required)

In the xy-plane, let C_1 be the circle $x^2+y^2=4$ and let C_2 be the ellipse $\dfrac{x^2}{2}+\dfrac{y^2}{6}=1$. Find the volume V of the solid formed by revolving the region of C_1 and C_2 in common (shaded part) about the y-axis.

(*Measurement skill*)