3rd Kyu

Section 2: Application Test

数学検定

PROFICIENCY TEST
IN
PRACTICAL MATHEMATICS

Test Time : 60 minutes

Test Instructions

1. Make sure that you have the correct level (Kyu) test.
2. Do not open the booklet until you are told to do so.
3. Write your examinee number and name on this page.
4. Write your name, examinee number and other necessary information on the answer sheet.
5. Write your answers on the answer sheets provided. Follow any instructions given when solving the problems.
6. If your answer contains a fraction, write the fraction in simplest form by reducing it to lowest terms.
7. If your answer contains a radical, write your answer in simplest radical form. For example, $\sqrt{12}$ must be expressed as $2\sqrt{3}$.
8. You may use a calculator.
9. Turn off your cell phone and do not use it during the test.
10. Ask an examination supervisor if your problem sheets have inconsistent page numbering or missing pages.
11. It is prohibited to disclose the problems to the general public, such as on the Internet, without permission.

Examinee Number	–	Name	

※Your personal information will be handled appropriately according to the "Handling of Personal Information" agreement that was approved at the time of registration.

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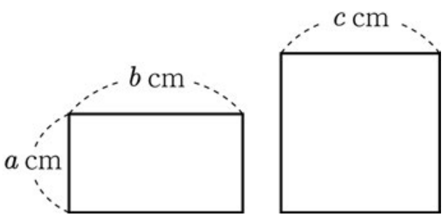
日本数学検定協会
The Mathematics Certification Institute of Japan

[3rd Kyu] Section 2: Application Test

1 The figure shows a rectangle of length b cm and width a cm and a square with sides of length c cm.



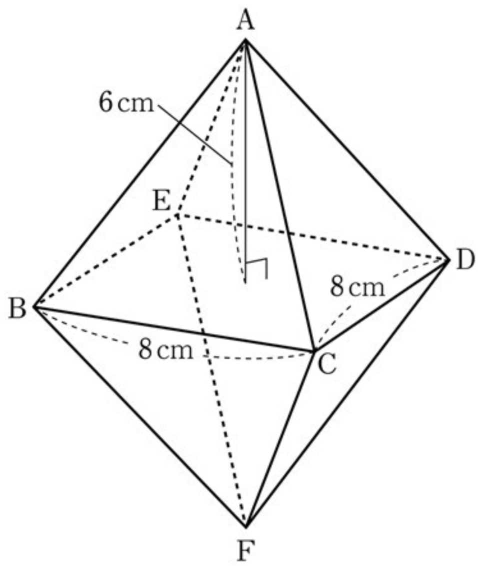
(1) Express the perimeter, in cm, of the rectangle in terms of a and b. (*Expression skill*)

(2) The sum of two times the area of the rectangle and the area of the square is less than 150 cm^2. Which of the following expressions represents the relationship? Choose one from ① to ⑥.

① $2ab + c^2 > 150$ ② $2ab + c^2 \geq 150$ ③ $2ab + c^2 < 150$

④ $2ab + c^2 \leq 150$ ⑤ $a^2b^2 + c^2 < 150$ ⑥ $a^2b^2 + c^2 \leq 150$

2 Octahedron ABCDEF is composed of two square-based pyramids with base length 8 cm and height 6 cm, with their square base touching.



(3) Find all edges that are neither parallel nor intersect with edge CD.

(4) Find the volume, in cm^3, of the octahedron. Include units in your answer. (*Measurement skill*)

3

Consider the graphs of the following functions from ① to ⑥.

① $y = 3x$ ② $y = -3x$ ③ $y = \frac{1}{3}x$

④ $y = -\frac{1}{3}x$ ⑤ $y = \frac{3}{x}$ ⑥ $y = -\frac{3}{x}$

(5) Choose all functions from ① to ⑥ whose graphs pass through the point $(-1, 3)$.

(6) Choose all functions from ① to ⑥ whose graphs are hyperbolas.

4

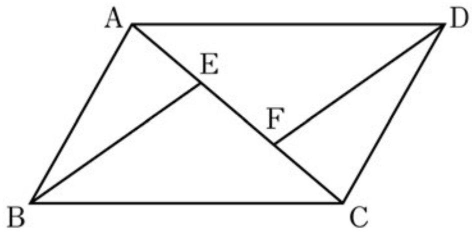
A box contains three red balls, two white balls and one black ball. Balls are randomly picked from the box.

(7) When one ball is picked, what is the probability of getting a white ball?

(8) When two balls are picked simultaneously, what is the probability of getting two red balls?

(9) When two balls are picked simultaneously, what is the probability of getting two balls of different colors?

5 In the figure of parallelogram ABCD, take points E and F on diagonal AC such that $AE = EF = FC$. When drawing line segments BE and DF, $BE = DF$ is proven as follows.



Proof

In $\triangle$ABE and $\triangle$CDF, we have

$AE = CF$ (given)$\cdots$(*).

Since (X), $AB = CD$ $\cdots$(**).

Since (Y) for $AB \parallel DC$, $\angle BAE = \angle DCF$ $\cdots$(***).

From (*), (**) and (***), since (Z), we have $\triangle ABE \cong \triangle CDF$.

Since the corresponding sides of congruent figures are equal, we have $BE = DF$.

(10) Which of the following sentences goes in X and Y? Choose one each from ① to ⑤.

① the opposite sides of a parallelogram are equal

② the opposite angles of a parallelogram are equal

③ the diagonals of a parallelogram bisect each other

④ the corresponding angles are equal

⑤ the alternate interior angles are equal

(11) Which of the following conditions in words goes in Z? Choose one from ① to ⑤.

① all sides are equal to their corresponding sides (SSS)

② two sides and the included angle are equal to their corresponding parts (SAS)

③ two angles and the included side are equal to their corresponding parts (ASA)

④ the hypotenuse and an acute angle are equal to their corresponding parts (HA)

⑤ the hypotenuse and a leg are equal to their corresponding parts (HL)

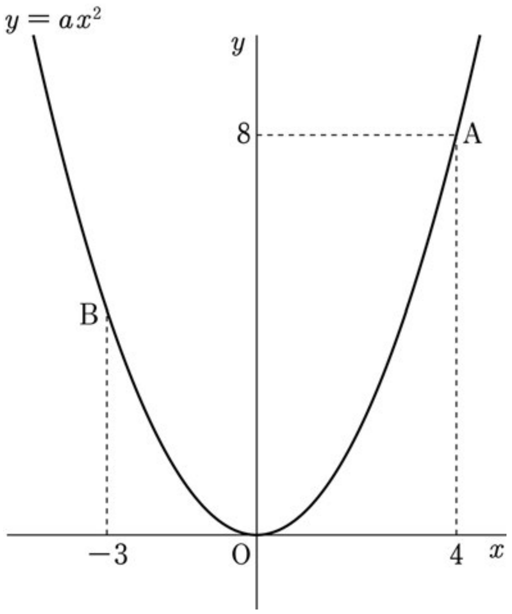
(12) If the area of $\triangle$ABE is 12cm^2, find the area, in cm^2, of $\triangle$ACD. Include units in your answer.

6 Answer the following.

(13) Let n be a positive integer. Find the minimum value of n such that $\sqrt{120n}$ is a positive integer.

(14) If $x=\sqrt{6}+\sqrt{2}$ and $y=\sqrt{6}-\sqrt{2}$, find the value of x^2-y^2.

7 Two points A and B lie on the graph of the function $y=ax^2$. The coordinates of point A are (4, 8) and the x-coordinate of point B is -3.

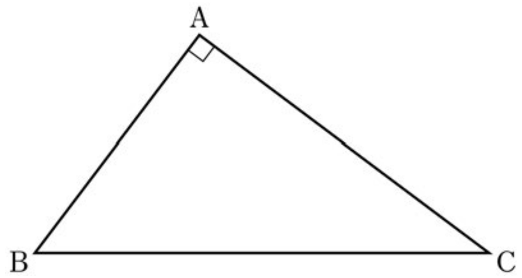


(15) Find the value of a. Write the steps leading to your answer.

(16) Find the coordinates of point B.

(17) Find the range of y for $-3 \le x \le 4$.

8 The figure shows right-angled triangle ABC with $\angle A = 90°$.



(18) Construct point P on side BC such that $\triangle ABC \sim \triangle PBA$. Follow the <Notes> below. You may also explain the procedure in words instead of actually constructing it. (*Construction skill*)

<Notes>

1. Use a compass and ruler for your construction. However, only use the ruler to draw straight lines.
2. Draw precisely how the compass arcs were drawn. Place a dot (•) at the position of the compass point.
3. Do not erase lines and/or arcs that are used for the construction and assign numbers ①, ②, ③, … to show the order in which they were drawn.

9 Answer the following. (*Organizing skill*)

(19) In a school, there are 18 students in the first grade, 27 students in the second grade and 20 students in the third grade. A survey was conducted to see how long it takes for each student to get school from home. The mean of the first, second and third grades are 15.5 minutes, 32.0 minutes and 21.5 minutes, respectively. Find the mean, in minutes, over all the students in the three grades.

(20) If there are values that are thought to be unusual, called outliers, in the set of data, the mean is skewed by the outliers.
Alice thought of five positive integers in her mind and the mean of them was 2021. Find the maximum possible value that Alice could have thought of. Note that the same number can be used more than once in the five numbers.