2nd Kyu

Section 2：Application Test

数学検定

PROFICIENCY TEST
IN
PRACTICAL MATHEMATICS

Test Time : 90 minutes

Test Instructions

1．Make sure that you have the correct level (Kyu) test.
2．Do not open the booklet until you are told to do so.
3．Write your examinee number and name on this page.
4．Write your name, examinee number and other necessary information on the answer sheets.
5．Write your answers on the answer sheets（No. 1, No. 2 and No. 3）. Write the steps leading to your answer. However if there are specific instructions for a problem, follow the instructions.
6．Problems 1 to 5 are selective problems. Choose three problems from the selective problems and fill in ⬯ to indicate which problems you chose. Then write your answers. Note that all of your answers will not be marked if you answered more than three problems from the selective problems. Problems 6 and 7 are required problems.
7．You may use a calculator.
8．Turn off your cell phone and do not use it during the test.
9．Ask an examination supervisor if your problem sheets have inconsistent page numbering or missing pages.
10. It is prohibited to disclose the problems to the general public, such as on the Internet, without permission.

Examinee Number	—	Name	

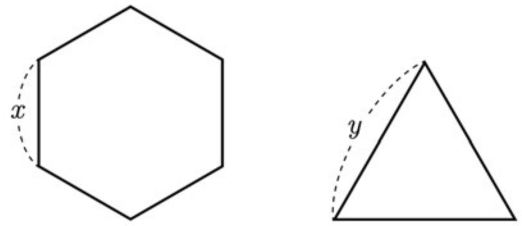
※Your personal information will be handled appropriately according to the "Handling of Personal Information" agreement that was approved at the time of registration.

公益財団法人
日本数学検定協会
The Mathematics Certification Institute of Japan

[2nd Kyu] Section 2: Application Test

1 (Selective)

The figure shows regular hexagon A with sides of length x and equilateral triangle B with sides of length y. Let S be the sum of the areas of A and B.



Regular hexagon A   Equilateral triangle B

(1) Express S in terms of x and y. Write only your answer.
(*Expression skill*)

(2) The sum of the perimeters of A and B is a constant value L. Find the minimum value of S and the corresponding value of $\dfrac{y}{x}$.

2 (Selective)

In a game, a player has a chance to get one of three items, A, B and C. Each item has a level of rarity (★★★ to ★) that indicates how unlikely it is for a player to get it (★★★ is the rarest). The table shows the probability of getting each item. Note that the probabilities are the same for each chance.

Item	Rarity	Probability
A	★★★	$\frac{1}{20}$
B	★★	$\frac{1}{5}$
C	★	$\frac{3}{4}$

(1) When a player has five chances successively to get items, what is the probability that the player gets at least one item A or B?

(2) When a player has six chances successively to get items, what is the probability that the player gets three item Cs, two item Bs and one item A?

3 (Selective)

Region D in the xy-plane is defined by

$$\begin{cases} x \geq 0 \\ y \geq 0 \\ y \leq -2x+6 \\ y \leq -\dfrac{2}{3}x+4 \end{cases}$$

(1) Sketch region D. Write only your answer. (*Expression skill*)

(2) Find the maximum value of $x+y$ with their corresponding values of x and y when the point (x, y) moves in region D.

4 (Selective)

An arithmetic sequence $\{a_n\}$ has first term 18 and common difference 12. Let b_n be the sum of the first n terms of the sequence $\{a_n\}$. (*Expression skill*)

(1) Find the nth term, denoted by b_n, of the sequence $\{b_n\}$.

(2) Find the sum of the first n terms, denoted by S_n, of the sequence $\{b_n\}$ and express it in factored form.

5 (Selective)

Given a positive integer n, a positive integer N is obtained by performing the following operation.

> ① If n is even, N is obtained by multiplying n by $\frac{3}{2}$.
>
> ② If n is an integer that leaves a remainder of 1 when dividing by 4, N is obtained by dividing $n-1$ by 4, and then multiplying the result by 3, and adding 1.
>
> ③ If n is an integer that leaves a remainder of 3 when dividing by 4, N is obtained by dividing $n+1$ by 4, and then multiplying the result by 3, and subtracting 1.

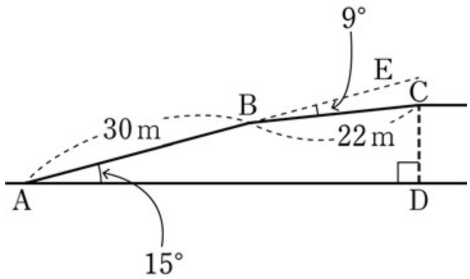
Replace the obtained number N with n and perform the operation repeatedly.
For example, starting with $n=17$, the number 12 is obtained by performing the operation six times as follows:

$$17 \xrightarrow{②} 13 \xrightarrow{②} 10 \xrightarrow{①} 15 \xrightarrow{③} 11 \xrightarrow{③} 8 \xrightarrow{①} 12$$

Starting with $n=70$, the number 70 is obtained again by performing the operation several times. Find the least number of operations to obtain the number 70. Also, find the maximum and minimum positive integers obtained during this process. Write only your answer. (*Organizing skill*)

6 (Required)

The figure shows a ramp with two different connected slopes in which the length of slope AB is 30 m and the length of slope BC is 22 m, $\angle DAB = 15^\circ$ and $\angle CBE = 9^\circ$ ($\angle CBE$ is the angle between straight lines AB and BC.)
Find the height, denoted by CD, in m, of the ramp. Use the values in the following table.
Round your answer off to one decimal place.
(*Measurement skill*)



θ	$\sin\theta$	$\cos\theta$
1°	0.0175	0.9998
2°	0.0349	0.9994
3°	0.0523	0.9986
4°	0.0698	0.9976
5°	0.0872	0.9962

θ	$\sin\theta$	$\cos\theta$
6°	0.1045	0.9945
7°	0.1219	0.9925
8°	0.1392	0.9903
9°	0.1564	0.9877
10°	0.1736	0.9848

θ	$\sin\theta$	$\cos\theta$
11°	0.1908	0.9816
12°	0.2079	0.9781
13°	0.2250	0.9744
14°	0.2419	0.9703
15°	0.2588	0.9659

7 (Required)

Find the quadratic function $f(x)$ that satisfies the following conditions ① and ② simultaneously.

① The following equality holds for $f(x)$.

$$(x-1)f'(x) = 2f(x) + x - 3$$

② $f'(2) = 3$