1

(1) (Answer) $S=\dfrac{\sqrt{3}}{4}(6x^2+y^2)$

(2) Since the sum of the perimeters of A and B is a constant value L, we have

$$6x+3y=L$$
$$y=-2x+\frac{L}{3}. \quad (*)$$

Substituting (*) into $S=\dfrac{\sqrt{3}}{4}(6x^2+y^2)$ gives

$$\begin{aligned}S&=\frac{\sqrt{3}}{4}\left\{6x^2+\left(-2x+\frac{L}{3}\right)^2\right\}\\&=\frac{\sqrt{3}}{4}\left(10x^2-\frac{4L}{3}x+\frac{L^2}{9}\right)\\&=\frac{5\sqrt{3}}{2}\left(x^2-\frac{2L}{15}x\right)+\frac{\sqrt{3}}{36}L^2\\&=\frac{5\sqrt{3}}{2}\left(x-\frac{L}{15}\right)^2-\frac{\sqrt{3}}{90}L^2+\frac{\sqrt{3}}{36}L^2\\&=\frac{5\sqrt{3}}{2}\left(x-\frac{L}{15}\right)^2+\frac{\sqrt{3}}{60}L^2.\end{aligned}$$

Hence, for $0<x<\dfrac{L}{6}$, S has the minimum value

$\dfrac{\sqrt{3}}{60}L^2$ at $x=\dfrac{L}{15}$.

From (*), we have $y=\dfrac{L}{5}$. Therefore, we obtain

$$\frac{y}{x}=3.$$

(Answer) Minimum value of S is $\dfrac{\sqrt{3}}{60}L^2$, $\dfrac{y}{x}=3$

2

(1) When a player has five chances successively, let A be the event “the player gets five items Cs”.
Then, the event “the player gets at least one item A or B” is the complementary event of A, which is $\overline{A}$.
The probability of occurring event A, denoted by $P(A)$, is

$$P(A)=\left(\frac{3}{4}\right)^5=\frac{243}{1024}.$$

Hence, the required probability $P(\overline{A})$ is

$$P(\overline{A})=1-P(A)$$
$$=1-\frac{243}{1024}$$
$$=\frac{781}{1024}.$$

(Answer) $\frac{781}{1024}$

(2) When a player has six chances successively, the number of ways of getting three items Cs, two items Bs and one item A is given by

$${}_6C_3\cdot{}_3C_2 \text{ ways.}$$

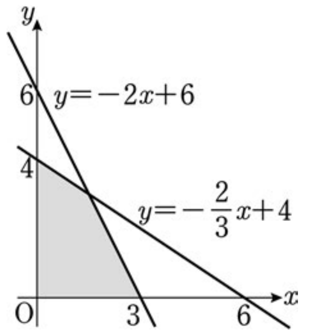
Hence, the required probability is

$${}_6C_3\cdot{}_3C_2\cdot\left(\frac{3}{4}\right)^3\cdot\left(\frac{1}{5}\right)^2\cdot\frac{1}{20}$$
$$=20\cdot 3\cdot\frac{27}{64}\cdot\frac{1}{25}\cdot\frac{1}{20}$$
$$=\frac{81}{1600}.$$

(Answer) $\frac{81}{1600}$

3

(1) In the figure, region D is the shaded part including the boundary lines.

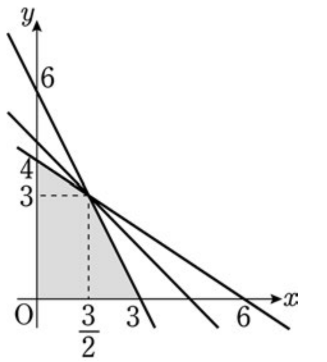


(2) Letting $x+y=k$, we have

$y=-x+k$, (*)

which represents a line with slope -1 and the y-intercept k.

Since the slope of the line $y=-2x+6$ is -2 and the slope of the line $y=-\frac{2}{3}x+4$ is $-\frac{2}{3}$, k has the maximum value if (*) passes through the point of intersection $\left(\frac{3}{2},\ 3\right)$ of the two lines $y=-2x+6$ and $y=-\frac{2}{3}x+4$.



Therefore, k has the maximum value $\frac{9}{2}$ at $x=\frac{3}{2}$ and $y=3$.

(Answer) Maximum value $\frac{9}{2}$ at $x=\frac{3}{2}$ and $y=3$

4

(1) $a_n = 18 + 12(n-1) = 12n + 6$.

Hence, we have

$$
\begin{aligned}
b_n &= \sum_{k=1}^{n} (12k+6) \\
&= 12 \cdot \frac{n(n+1)}{2} + 6n \\
&= 6n^2 + 12n.
\end{aligned}
$$

(Answer) $b_n = 6n^2 + 12n$

---

(2) Since $b_n = 6n^2 + 12n$, we have

$$
\begin{aligned}
S_n &= \sum_{k=1}^{n} (6k^2 + 12k) \\
&= 6 \cdot \frac{n(n+1)(2n+1)}{6} + 12 \cdot \frac{n(n+1)}{2} \\
&= n(n+1)(2n+1) + 6n(n+1) \\
&= n(n+1)\{(2n+1)+6\} \\
&= n(n+1)(2n+7).
\end{aligned}
$$

(Answer) $S_n = n(n+1)(2n+7)$

5	(Answer) The least number of operations 12. Maximum integer 111, minimum integer 44.

6

Drawing a line from point B perpendicularly to line segment CD, we let H be the point of intersection.

Since

$$\angle \mathrm{HBE} = \angle \mathrm{DAB} = 15°,$$

we have

$$\angle \mathrm{HBC} = \angle \mathrm{HBE} - \angle \mathrm{CBE} = 6°.$$

Drawing a line from point B perpendicularly to line segment AD and letting I be the point of intersection, $\mathrm{BI} = \mathrm{HD}$.

Therefore, the required height of the ramp is

$$\begin{aligned} \mathrm{CD} &= \mathrm{CH} + \mathrm{HD} \\ &= \mathrm{CH} + \mathrm{BI} \\ &= \mathrm{BC}\sin 6° + \mathrm{AB}\sin 15° \\ &= 22 \cdot 0.1045 + 30 \cdot 0.2588 \\ &= 10.063 \\ &\approx 10.1 \ \text{(m)}. \end{aligned}$$

(Answer) 10.1 m

7

Let

$$f(x)=ax^2+bx+c\,, \quad ③$$

where $a\,(\,a\neq 0\,)$, b and c are constants. Then we have

$$f'(x)=2ax+b\,. \quad ④$$

Substituting ③ and ④ into $(x-1)f'(x)=2f(x)+x-3$ gives

$$(x-1)(2ax+b)=2(ax^2+bx+c)+x-3$$
$$2ax^2+bx-2ax-b=2ax^2+2bx+2c+x-3$$
$$(2a+b+1)x+(b+2c-3)=0.$$

Since this must be an identity for x, equating the coefficients on both sides, we have

$$2a+b+1=0\,, \quad ⑤$$
$$b+2c-3=0\,. \quad ⑥$$

From ② and ④, we have

$$4a+b=3\,. \quad ⑦$$

Solving the system of equations ⑤ and ⑦ gives

$$a=2\,, \quad b=-5\,.$$

Substituting $b=-5$ into ⑥ gives

$$c=4\,.$$

Therefore, the required quadratic function is

$$f(x)=2x^2-5x+4\,.$$

(Answer) $f(x)=2x^2-5x+4$