1st Kyu

Section 1 : Calculation Test

数学検定

PROFICIENCY TEST
IN
PRACTICAL MATHEMATICS

Test Time : 60 minutes

Test Instructions

1．Make sure that you have the correct level (Kyu) test.

2．Do not open the booklet until you are told to do so.

3．Write your examinee number and name on this page.

4．Write your name, examinee number and other necessary information on the answer sheets.

5．Write only answers on the answer sheets provided.

6．You may not use a calculator, ruler or compass.

7．Turn off your cell phone and do not use it during the test.

8．Ask an examination supervisor if your problem sheets have inconsistent page numbering or missing pages.

9．It is prohibited to disclose the problems to the general public, such as on the Internet, without permission.

Examinee Number	–	Name	

※Your personal information will be handled appropriately according to the "Handling of Personal Information" agreement that was approved at the time of registration.

公益財団法人
日本数学検定協会
The Mathematics Certification Institute of Japan

[1st Kyu] Section 1: Calculation Test

1 Determine the value of the complex number a such that the following quartic equation has multiple roots. Use i for the imaginary unit.

$$x^4 - ax^3 - (4a^2 - 2a + 6)x^2 + (4a^3 + 2a^2 - 6a)x - 4a^3 + 12a^2 = 0$$

2 Find the sum of the following and express it in factored form with integer coefficients.

$$\sum_{k=1}^{n} (6k+3)\, k^2 (k+1)^2$$

3 The probability density function $f(x)$ of random variable X is given by

$$f(x) = \begin{cases} x^2, & 0 \le x \le 1; \\ -\dfrac{3}{4}x + \dfrac{7}{4}, & 1 < x \le \dfrac{7}{3}; \\ 0, & x < 0 \ \text{ or } \ \dfrac{7}{3} < x. \end{cases}$$

Find the expected value of X, denoted by $E[X]$.

4 Answer the following.

① For $\boldsymbol{d}_1 = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$ and $\boldsymbol{d}_2 = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix}$, find the outer product (vector product) $\boldsymbol{d}_1 \times \boldsymbol{d}_2$.

② In the xyz-space, find the distance between the following two lines ℓ_1 and ℓ_2.

$$\ell_1 : \frac{x-1}{3} = \frac{y+5}{2} = \frac{z}{4} \quad \text{and} \quad \ell_2 : \frac{x+4}{3} = \frac{y-2}{5} = z+1$$

5 Consider $f(x) = \operatorname{Arctan} x$. Note that $\operatorname{Arctan} x$ represents the inverse function of $\tan x$ that satisfies $-\frac{\pi}{2} < \operatorname{Arctan} x < \frac{\pi}{2}$.

① Find the derivative function $f'(x)$.

② For the second derivative $f''(x)$, find $f''(-\sqrt{3})$.

6 Calculate the following determinant and express it in expanded form.

$$\begin{vmatrix} 1 & 2x & 3x^2 & 4x^3 \\ 2x & 3x^2 & 4x^3 & 1 \\ 3x^2 & 4x^3 & 1 & 2x \\ 4x^3 & 1 & 2x & 3x^2 \end{vmatrix}$$

7 Evaluate the following double integral for the region $D = \{(x, y) \mid x^2 \leq y \leq x\}$ in the xy-plane.

$$\iint_D (7x^2y + 12xy^2)dxdy$$