1

(1) From $x^2+(a^2+a+1)x+a^2+a<0$, $\cdots$①

$(x+a^2+a)(x+1)<0$. If $-a^2-a=-1$, there is no real number x that satisfies ①. We consider different cases comparing $-a^2-a$ and -1.

If $-a^2-a<-1$ $\cdots$②, the solution of ① is

$-a^2-a<x<-1$.

The condition that the only one integer satisfies this range is $-3\le -a^2-a<-2$. $\cdots$③

The intersection of ② and ③ is ③. Next, we solve ③, that is,

$a^2+a-2>0$ $\cdots$④ and $a^2+a-3\le 0$ $\cdots$⑤

Since $a^2+a-2=(a+2)(a-1)$, the solution of ④ is

$a<-2,\ 1<a$ $\cdots$④'

Solving $a^2+a-3=0$ for a gives $a=\dfrac{-1\pm\sqrt{13}}{2}$.

Thus, the solution of ⑤ is

$\dfrac{-1-\sqrt{13}}{2}\le a\le\dfrac{-1+\sqrt{13}}{2}$ $\cdots$⑤'

Since the solution of ③ is the intersection of ④' and ⑤', we have

$\dfrac{-1-\sqrt{13}}{2}\le a<-2,\ 1<a\le\dfrac{-1+\sqrt{13}}{2}$. $\cdots$③'

If $-1<-a^2-a$ $\cdots$⑥, the solution of ① is

$-1<x<-a^2-a$.

The condition that the only one integer satisfies this range is $0<-a^2-a\le 1$. $\cdots$⑦

The intersection of ⑥ and ⑦ is ⑦. Next, we solve ⑦, that is,

$a^2+a<0$ $\cdots$⑧ and $a^2+a+1\ge 0$ $\cdots$⑨

Since $a^2+a=a(a+1)$, the solution of ⑧ is

$-1<a<0$. $\cdots$⑧'

Since $a^2+a+1=\left(a+\dfrac{1}{2}\right)^2+\dfrac{3}{4}\ge\dfrac{3}{4}$, the solution of ⑨ is all real numbers. Hence the solution of ⑦ is ⑧'.

Therefore, the required range is the union of ③' and ⑧', that is,

$\dfrac{-1-\sqrt{13}}{2}\le a<-2,\ -1<a<0,\ 1<a\le\dfrac{-1+\sqrt{13}}{2}$.

(Answer) $\dfrac{-1-\sqrt{13}}{2}\le a<-2,\ -1<a<0,$

$1<a\le\dfrac{-1+\sqrt{13}}{2}$.

2

Let A be the event

$(a-b)(b-c)(c-d)(d-e)(e-f)\ne 0$.

Then the event

$(a-b)(b-c)(c-d)(d-e)(e-f)=0$

is the complement of A, written $\overline{A}$.

The event A is the event that "the numbers facing up from the 2nd to 6th roll are different from each of the previous number."

Thus,

$P(A)=\left(\dfrac{5}{6}\right)^5=\dfrac{3125}{7776}$.

The required probability is

$P(\overline{A})=1-P(A)=\dfrac{4651}{7776}$.

(Answer) $\dfrac{4651}{7776}$

3

We find the fourth power of $z=a+i$.

$z^2=(a+i)^2=a^2-1+2ai$,

$z^4=(a^2-1+2ai)^2$

$=(a^2-1)^2-4a^2+4a(a^2-1)i.$

Since this is equal to the real number r,

$r=(a^2-1)^2-4a^2=a^4-6a^2+1$ $\cdots$①

$a(a^2-1)=0$ $\cdots$②

Solving ②,

$a(a+1)(a-1)=0$.

Thus,

$a=0,\ -1,\ 1$.

Substituting each value into ①, we have

$r=1$ when $a=0$

$r=-4$ when $a=-1$

$r=-4$ when $a=1$

(Answer) $(a,\ r)=(0,\ 1),(-1,\ -4),(1,\ -4)$

4

(1) Squaring $|\vec{a}-\vec{b}|$ gives

$$|\vec{a}-\vec{b}|^2=(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$$
$$=|\vec{a}|^2-2\,\vec{a}\cdot\vec{b}+|\vec{b}|^2.$$

Since $|\vec{a}|=|\vec{b}|=2$ and $\vec{a}\cdot\vec{b}=1-\sqrt{5}$ $\cdots$①,

$$|\vec{a}-\vec{b}|^2=4-2(1-\sqrt{5})+4$$
$$=6+2\sqrt{5}$$
$$=(1+\sqrt{5})^2$$

Since $|\vec{a}-\vec{b}|>0$, $|\vec{a}-\vec{b}|=1+\sqrt{5}$

(Answer) $1+\sqrt{5}$

(2) We find the value of t that satisfies $|\vec{a}+t\vec{b}|=|\vec{a}-\vec{b}|$. Using ①, squaring the left hand side,

$$|\vec{a}|^2+2t\,\vec{a}\cdot\vec{b}+t^2\,|\vec{b}|^2$$
$$=4t^2+2(1-\sqrt{5})t+4$$

From (1), $4t^2+2(1-\sqrt{5})t+4=6+2\sqrt{5}$.

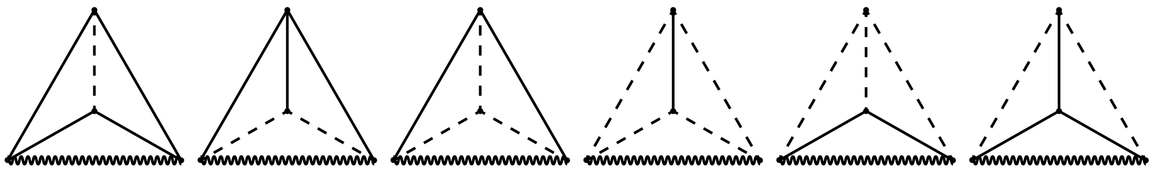
Solving it for t,

$$2t^2+(1-\sqrt{5})t-1-\sqrt{5}=0$$
$$(t+1)(2t-1-\sqrt{5})=0$$

Therefore, $t=-1,\ \dfrac{1+\sqrt{5}}{2}$.

(Answer) $t=-1,\ \dfrac{1+\sqrt{5}}{2}$

5



6

$$\frac{1}{S}\left(\frac{\sin A}{a}+\frac{\sin B}{b}+\frac{\sin C}{c}\right)$$
$$=\frac{1}{a}\cdot\frac{\sin A}{S}+\frac{1}{b}\cdot\frac{\sin B}{S}+\frac{1}{c}\cdot\frac{\sin C}{S}\quad\cdots①$$

Since

$$S=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B=\frac{1}{2}ab\sin C,$$
$$\frac{\sin A}{S}=\frac{2}{bc},\quad \frac{\sin B}{S}=\frac{2}{ca},\quad \frac{\sin C}{S}=\frac{2}{ab}.$$

Substituting these values into ①,

$$\frac{1}{S}\left(\frac{\sin A}{a}+\frac{\sin B}{b}+\frac{\sin C}{c}\right)$$
$$=\frac{1}{a}\cdot\frac{2}{bc}+\frac{1}{b}\cdot\frac{2}{ca}+\frac{1}{c}\cdot\frac{2}{ab}$$
$$=\frac{6}{abc}$$

(Answer) $\dfrac{6}{abc}$

7

The derivative function of $y=x^2-5x+3$ is $y'=2x-5$. Since the line ℓ is the tangent line at point A(2, -3), the equation is

$$y=(2\cdot2-5)(x-2)-3$$
$$=-x-1$$

(Answer) $y=-x-1$

Since $-x-1\le x^2-5x+3$ for $0\le x\le 2$, the required area S is

$$S=\int_0^2\{x^2-5x+3-(-x-1)\}dx$$
$$=\int_0^2(x^2-4x+4)dx$$
$$=\left[\frac{x^3}{3}-2x^2+4x\right]_0^2$$
$$=\frac{8}{3}-8+8-0$$
$$=\frac{8}{3}$$

(Answer) $S=\dfrac{8}{3}$