1st Kyu

Section 2 : Application Test

数学検定

PROFICIENCY TEST
IN
PRACTICAL MATHEMATICS

Test Time : 120 minutes

Test Instructions

1. Make sure that you have the correct level (Kyu) test.
2. Do not open the booklet until you are told to do so.
3. Write your name and examinee number on this page.
4. Write your name, examinee number and other necessary information on the answer sheets.
5. Write your answers on the answer sheets (they are numbered 1 through 4). Write the steps leading to your answer. However if there are specific instructions for a problem, follow the instructions.
6. Problems 1 to 5 are selective problems.
   Choose two problems from the selective problems and fill in ◯ to indicate which problems you chose. Then write your answers. Note that all of your answers will not be marked if you answered more than two problems from the selective problems.
   Problems 6 and 7 are required problems.
7. You may use a calculator.
8. Turn off your cell phone and do not use it during the test.
9. Ask an examination supervisor if your problem sheets have inconsistent page numbering or missing pages.
10. It is prohibited to disclose the problems to the general public, such as on the Internet, without permission.

Name	
Examinee Number	–

公益財団法人
日本数学検定協会
The Mathematics Certification Institute of Japan

[1st Kyu]   Section 2: Application Test

1 (Selective)

For the following equation, find integer solutions (x, y) or prove that there are no integer solutions.

$$7x^2 - 9y^2 = 391$$

2 (Selective)

Consider the following function defined for all real numbers.

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & (x \neq 0) \\ 1 & (x = 0) \end{cases}$$

(1) Prove that $f(x)$ is differentiable for all real numbers x and $f'(x)$ is continuous, where $f'(x)$ is the derivative function of $f(x)$.

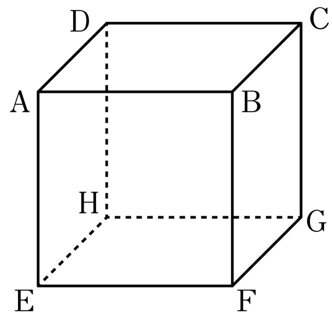
(2) A sequence $\{a_n\}$ is defined by $a_n = f'(n\pi)$ for $f'(x)$ in (1). Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} a_n$.

3 (Selective)

The figure on the right shows the cube ABCD-EFGH of edges of length ℓ. The cube can be moved in the xyz-space with fixed vertex A at the origin. Let $(x_1,\ y_1,\ z_1)$, $(x_2,\ y_2,\ z_2)$ and $(x_3,\ y_3,\ z_3)$ be the coordinates of the vertices B, D and E, respectively. Define

$$\alpha = x_1 + y_1 i,\ \ \beta = x_2 + y_2 i,\ \ \gamma = x_3 + y_3 i,$$

for each vertex, where i represents the imaginary unit.



(1) Prove that the complex number $\alpha^2+\beta^2+\gamma^2$ takes a constant value for any position of the vertices B, D and E. Also, find the value of $\alpha^2+\beta^2+\gamma^2$.

(2) Express ℓ in terms of α, β and γ.

4 (Selective)

Consider the following fact for the t-distribution, which is one of the probability distributions.

> If n samples x_k $(k = 1, 2, 3, \ldots, n)$ are randomly drawn from a normally distributed population with the mean μ and the standard deviation σ, we define the sample mean $\bar{x}$ and sample standard deviation s as
>
> $$\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x_k \quad \text{and} \quad s = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}(x_k - \bar{x})^2}\,.$$
>
> Then, $X = \dfrac{\bar{x} - \mu}{\dfrac{s}{\sqrt{n}}}$ follows the t-distribution with $n-1$ degrees of freedom.
>
> (Note that you don't need to prove this)

Using the fact above, we find the confidence interval of the weight mean m (g) of a product P. Use the values in the t-distribution table given. For the confidence interval, round up the least upper bound to four significant figures and round down the greatest lower bound to four significant figures.

(1) 10 sample products P are selected at random and their weights are the following.

15.19 g	14.78 g	14.89 g	15.11 g	15.05 g
14.79 g	15.16 g	14.85 g	14.94 g	15.24 g

Find the 95 % confidence interval of m.

(2) 200 sample products P are selected at random and measured their weights. If we have

Sample mean is 15.00 g and sample standard deviation is 0.250 g,

then find the 95 % confidence interval of m. Note that you may consider that the number 200 is large enough.

5 (Selective)

Kepler's first and second laws of planetary motion states

> 1st law: The orbit of a planet in our solar system is an ellipse with the Sun at one of its two foci.
>
> 2nd law: The line segment joining the Sun to a planet sweeps out equal areas in equal times.

Suppose that a planet in our solar system moves according to the laws above. Let C be the elliptic orbit of the planet. Let P and P' be the positions of the planet at time t and $t+\Delta t$ ($\Delta t>0$), respectively. Let S be the Sun at one of the foci and let S' be the other focus.
If the value of Δt and the eccentricity e ($e>0$) is small enough, we may suppose the following.

> ① Point P' lies on the tangent line of C at P.
>
> ② $\mathrm{S'P'} \approx \mathrm{S'P}$
>
> ③ The value of e^2 is small enough to be ignored, that is, we may regard $e^2 \approx 0$.
>
> (You don't need to prove them.)

Under the conditions above, show that we can regard that the angular velocity of the planet around S' is constant (the measure of $\angle \mathrm{PS'P'}$ within the time duration Δt is proportional to Δt).

6 (Required)

Consider two $n\times n$ square matrices A and B, where n is an integer greater than or equal to 2.

(1) Let $\mathrm{tr}(M)$ be the sum of diagonal elements of $n\times n$ square matrix M. Prove that

$$\mathrm{tr}(AB) = \mathrm{tr}(BA).$$

(2) Find A^n if $AB-BA=A$. Note that you may use the following fact without its proof.

For the system of equations for n unknowns, x_1, x_2, x_3, ..., x_n,

$$\sum_{k=1}^{n} x_k = \sum_{k=1}^{n} x_k{}^2 = \sum_{k=1}^{n} x_k{}^3 = \cdots = \sum_{k=1}^{n} x_k{}^n = 0,$$

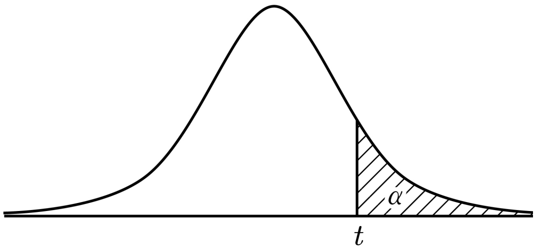
the only solution is $x_1 = x_2 = x_3 = \cdots = x_n = 0$.

7 (Required)

Find the general form for the function of three real variables $f(x, y, z)$ that satisfies all the following conditions.

$$\begin{cases} f(x, y, z) \text{ is a homogeneous polynomial of degree 3 (the polynomial whose nonzero terms all have the degree of 3).} \\ \dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} + \dfrac{\partial f}{\partial z} = 0 \\ \dfrac{\partial^2 f}{\partial x \partial y} + \dfrac{\partial^2 f}{\partial y \partial z} + \dfrac{\partial^2 f}{\partial z \partial x} = 0 \\ \dfrac{\partial^3 f}{\partial x \partial y \partial z} = 0 \end{cases}$$

The values for t-distribution with n degrees of freedom for a range of one tailed critical region α.



The t-Distribution Table

n \ α	0.20	0.10	0.05	0.025	0.010	0.005	0.001	0.0005
1	1.376	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	1.061	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.978	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.920	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.906	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.879	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.876	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.873	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.870	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.868	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.866	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.865	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.863	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.862	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.861	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.860	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.859	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.858	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.858	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.857	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.856	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.856	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.855	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.855	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.854	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.854	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.851	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.848	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.845	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	0.842	1.282	1.645	1.960	2.326	2.576	3.090	3.291