

[1st Kyu] Section 1: Calculation Test

- 1 Solve the following system of equations for real numbers x , y and z , giving all solutions.

$$\begin{cases} xy^2z^3 = \frac{12}{7} \\ x^3yz^2 = -\frac{7}{11} \\ x^2y^3z = -\frac{11}{12} \end{cases}$$

- 2 Simplify the following expression. Note that i represents the imaginary unit.

$$\frac{(1-i)^{11}}{(-\sqrt{3}+i)^6}$$

- 3 Find the inverse matrix of the following matrix.

$$\begin{pmatrix} 1 & 0 & 4 \\ 3 & -1 & 0 \\ -2 & 1 & -1 \end{pmatrix}$$

4 For a surface $z = \arctan \frac{y}{x}$, where $x \neq 0$ and $-\frac{\pi}{2} < z < \frac{\pi}{2}$, answer the following.

① Compute $\frac{\partial z}{\partial x}$.

② Find the equation of the tangent plane to the above surface at the point $(x, y, z) = \left(1, -1, -\frac{\pi}{4}\right)$.

5 A bag contains 8 white balls and 2 red balls. One ball is drawn from the bag at random on the 1st trial. If the ball is red, this is the end of the trial. If the ball is white, replace the ball in the bag and draw a ball at random on the 2nd trial. Repeat the trials until a red ball is drawn. Suppose that a red ball is drawn on the X th trial.

① Find the expected value of X .

② Find the expected value of X^2 .

6 Find the eigenvalues of the following matrix.

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \end{pmatrix}$$

7 Solve the differential equation

$$(3x - y) \frac{dy}{dx} = 2x,$$

under the initial condition “ $y = \frac{1}{2}$ when $x = 0$ ”.